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Network formation, statistical physics and social dynamics

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The focus of this project was to understand the formation and dynamics of complex networks with the methods of statistical physics. Some of our work was motivated by fundamental mathematical questions while others were driven by the analysis of real data.

I. The geography and carbon footprint of mobile phone use in Cote d'Ivoire

In 2013, the mobile phone provider Orange released a data base of mobile phone use in Cote d'Ivoire. These data provided a rare insight into the use of mobile technology in a developing country. We performed a series of spatial data analyses that reveal important geographic aspects of the country's mobile phone coverage. We first mapped the locations of base stations, which house the antennas, with respect to the population distribution and the number and duration of calls at each base station. We used a cartographic technique called "density-equalizing map" or cartogram that I developed in 2004 (PNAS, vol. 101, p. 7499ff.). This method allowed us to scale all subprefectures in Cote d'Ivoire such that the areas are proportional to the number of inhabitants (Fig. I).

Although the geographic distribution of the base stations is largely explained by the heterogeneous population distribution, the point pattern on the cartogram still identifies regions of significantly higher per-capita base station density. Especially in Abidjan the dots are noticeably aggregated. Because one development objective must be to provide a roughly equal per-capita number of base stations for the entire population of Cote d'Ivoire, sparser point densities on the cartogram highlight those regions on which expansion of the network should be focused in the future.

In further analyses, we estimated the energy consumed by the mobile phone network. We found that mobile networks in Cote d'Ivoire are likely to contribute a greater proportion to the national greenhouse gas emissions than those in industrialized countries. However, the mobile network still only consumes 0.95% of Cote d'Ivoire's electrical energy and is therefore only of minor concern for the nation's CO_2 carbon footprint.

II. The development of the global network of cargo shipping

Cargo shipping is estimated to transport 90% of world trade so that, in economic terms, the routes of cargo ships form arguably the world's most important transport network. In previous work I had performed an analysis of the network based on data from 2007 exclusively. Thanks to a collaboration with César Ducruet (CNRS Paris) I now have access to cargo ship records for a longer period, going as far back as 1890. I used this Marie Curie Fellowship to begin data analysis, for the time being focusing on the distributions of vessel calls and the ports' degrees (i.e. the number of other ports that a port is directly linked with). Much previous work on economic and social networks has claimed that such centrality distributions in economic and social networks have a characteristic mathematical form: a power law, also sometimes called a "scale-free" distribution. Power laws have characteristically heavy tails, which means that some nodes are significantly better connected than the average.

Our work, however, demonstrates that a power law is inadequate to fit the cargo ship data. Instead we propose alternatives, such as lognormal or Weibull distributions, that perform consistently better for all years for which we have data. Cargo ship traffic has thus for the entire study period been heavy-tailed, but the distribution is not scale-free. We also calculated the Gini coefficient, a measure of inequality in economics. The Gini coefficient of port traffic has slightly, but statistically significantly, decreased over the study period, highlighting a tendency towards a more polycentric distribution in cargo shipping. These results are informative for economic forecasts and provide important lessons for successful planning of port expansions.

III. Opinion formation on a gradient

We model the formation of opinions, an example of a collective game-theoretic social behaviour, where individuals change their inherently preferred opinion if their friends disagree. Real preferences often depend on regional cultural differences, which we model as a spatial gradient g in the initial opinions. The presence of spatial gradients in real election results, especially between rural and urban populations, has been pointed out by many political scientists. However, the quantitative consequences for opinion formation have so far been unknown.

We approach opinion formation with the tools of statistical physics to analyze the percolation pattern (i.e. the geometry of connected clusters of like-minded opinions). We show that opinion clusters are typically in the standard (i.e. independent) percolation universality class. As a consequence, influencing each other's opinion usually only creates consensus on a local scale, but over long distances the opinions remain uncorrelated. Thus, opinions remain mixed if averaged over long length scales. However, we also present an alternative model where a sharp spatial division between opinions occurs.

Representative patterns for these two different scenarios are shown in Fig. 2. Black and white sites represent opposite opinions. The sites are initially distributed randomly, but with a slight gradient so that in each panel there is a majority of white sites on the left of the lattice and a majority of black sites on the right. The grey squares mark the front of the largest black cluster ("percolation hull"). In (a) the sites follow the majority vote rule (i.e. if three of four neighbouring sites have a different opinion, the focal site will switch its opinion). In (b) the sites follow the majority vote rule 80% of the time and otherwise choose a random opinion.

We demonstrate numerically and analytically how the interface widths, fractal dimensions and cluster size distributions differ between these two scenarios. The interface width w of the transition between opinions scales in the model depicted in (a) as $w \propto g^{4/7}$, but as $w \propto g^{1/4}$ in (b). The fractal dimension of the interface is $D_f = 7/4$ in (a), but $D_f = 1$ in (b), so that in the latter case the interface is not a fractal despite its roughness. The cluster size distribution in (a) exhibits the same asymptotic power law as independent percolation, while in (b) the distribution is not scale-free and thus consistent with first-order percolation. Our results settle a controversial debate in the literature about the universality class of percolation in "non-consensus opinion models": the model shown in (a) was previously claimed to differ from standard percolation (Shao et al., Phys. Rev. Lett., 103:018701, 2009), but our data convincingly demonstrates the opposite. However the model in (b) proves that with alternative rules for opinion formation non-standard percolation is conceivable.

IV. The Ising chain constrained to an even or odd number of positive spins

The work on opinion formation that I have just described raises the intriguing question of a general equation-based solution for those models. It turns out that, after some mathematical transformations, the one-dimensional models can indeed be mapped onto a classic problem in statistical physics: the Ising model. In the Ising model, sites are occupied by so-called spins that can point either up or down. The total energy in the Ising model is determined by the number of neighbours pointing in opposite directions. The mapping from opinion formation to Ising model required one additional twist that had not been previously studied: the number of positive spins is constrained to be either odd or even, depending on whether the total number of individuals in the opinion formation model is an odd or even number.

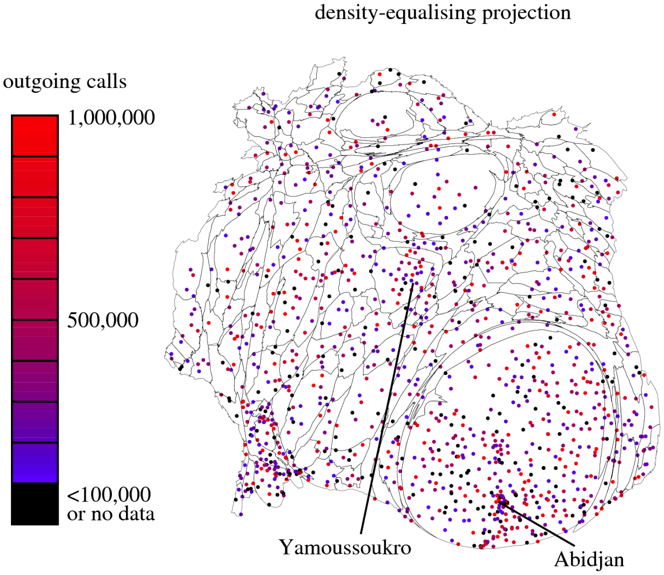

Fig. 1. Cartogram of Cote d'Ivoire where all subprefectures are scaled so that their areas are proportional to the number of inhabitants. The dots indicate the locations of base stations operated by Orange. The colours of the dots indicate the number of outgoing calls at each base station.

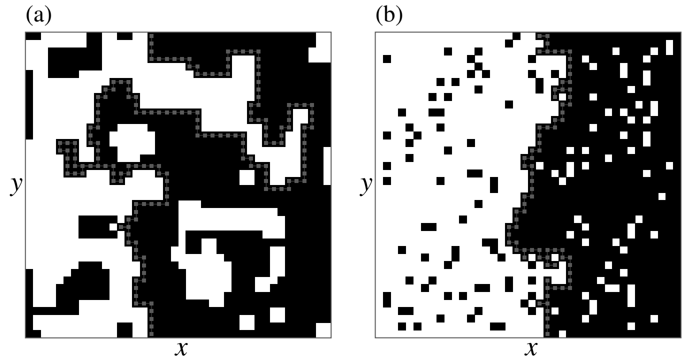

Fig. 2. Two examples of opinion formation on a gradient.

This observation called for a thorough investigation of the problem. In a recent publication I calculated the partition function using a generalization of the transfer matrix method. On this basis, I derived the exact magnetization, susceptibility, internal energy, heat capacity and correlation function. I showed that in general the constraints substantially slow down convergence to the thermodynamic limit (i.e. the limit of infinitely many nodes in the network). By taking the thermodynamic limit together with the limit of zero temperature and zero magnetic field, the constraints lead to new scaling functions and different probability distributions for the magnetization.

This work is a starting point for investigating the more general case of opinions on two-dimensional or even more complex social networks. It can also be generalized to more complicated social dynamics. Preliminary results show that the matching-pennies game, a textbook example of a two-player game with no pure strategy Nash equilibrium, can be mapped to the Ising model.

V. Continuing work

Towards the end of my Marie Curie Fellowship I began collaborating with Dr. Géza Ódor (MTA-TTK Budapest) on analyzing data from the Open Connectome project. From the website (www.openconnectomeproject.org) we obtained network data for the human brain where nodes are voxels in MRI scans and edges are fibre tracts. The same statistical techniques that I had developed for the analysis of cargo ship routes (Sec. II) can be successfully applied to the brain. Our results may open the path towards more realistic models of brain functions and dynamics.