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Cartogram

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Synonyms

Anamorphic maps

Definition

Cartograms are thematic maps in which the sizes of geographic objects appear in proportion to a quantitative mapping variable. There are two types of cartograms. If the mapping variable is represented by the areas of the depicted regions, the map is called an *area cartogram*. If distances between points are a substitute for the mapping variable, the map is called a *distance cartogram*. Both cartogram types visualize quantitative data by distorting familiar geographic features. However, area cartograms and distance cartograms have distinct applications, and their construction requires different mathematical and algorithmic techniques. Suitable mapping variables for area cartograms are quantities that can be associated with nonoverlapping regions on a map (e.g., countries or provinces). Area cartograms are frequently used in human geography to visualize population distributions ("isodemographic maps") or economic indicators (e.g., gross domestic product). Distance cartograms usually represent travel times ("time-space maps"), but other measures of separation (e.g., travel costs or fuel consumption) can also be used.

Area Cartogram

The mathematical objective of an area cartogram (also known as "value-by-area map") is to transform geographic regions, given in the form of multipolygons, into new multipolygons with areas that match quantitative data associated with each region. The output should maintain certain properties of conventional geographic maps (e.g., the locations and shapes of regions) and maintain the regions' topology (i.e., regions should share a common border on the cartogram if and only if they share a common border in geographic space). It is generally impossible to achieve all these objectives simultaneously. A variety of area cartogram types have been developed that relax different constraints. The main categories of area cartograms are rectangular cartograms, mosaic cartograms, circular cartograms, noncontiguous cartograms, and contiguous cartograms (Fig. 1). For a review of area cartograms, including further cartogram types, see Tobler (2004), and Nusrat and Kobourov (2016).

Rectangular Cartograms

In a rectangular cartogram, each region is represented by a rectangle with an area equal to a numeric input (Fig. 1a). Rectangular cartograms are the oldest cartogram type with hand-drawn examples dating back to the late nineteenth century (see Tobler 2004 for a historical account). Maintaining the correct adjacency of regions is generally impossible in rectangular cartograms. Algorithms have been created to automate the construction of rectangular cartograms with correct topology, but only after landlocked regions with fewer than four neighbors are manually merged with one of the surrounding regions (Buchin et al. 2012). A variation of the rectangular cartogram with fewer topological constraints is the rectilinear cartogram, in which each region can be composed of multiple rectangles.

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Mosaic Cartograms

Mosaic cartograms divide the map into a set of small square-shaped or hexagonal tiles, each of equal size and orientation (Fig. 1b). The most suitable thematic mapping variables for mosaic cartograms are those that take small integer values so that one tile corresponds to one discrete unit (e.g., a seat in parliament). Tiles are assigned to regions such that contiguous geographic regions are represented by contiguous sets of tiles. For some data, topological errors are inevitable because the details of real geographic boundaries cannot be represented by the discrete shapes of the tiles. To minimize the topological errors, Dorling (1996) developed a cellular automaton algorithm for the construction of mosaic cartograms. A recently proposed algorithm aims to improve the preservation of the regions' shapes (Cano et al. 2015).

Circular Cartograms

Circular cartograms represent each region by a circle with an area that is proportional to the thematic mapping variable (Fig. 1c). Circles are placed such that they may touch, but do not overlap. Dorling's (1996) circular cartogram algorithm and a recent refinement by Inoue (2011) aim to place the centers of the circles approximately at the same position where the region centroids would be on a conventional map. Unlike rectilinear and mosaic cartograms, circular cartograms always leave unfilled space in the interior of the map. The restriction to circular shapes may also force neighboring geographic regions to appear separated on the cartogram.

Noncontiguous Cartograms

Noncontiguous cartograms deliberately introduce empty space between the depicted regions (Fig. 1d) and, thus, make no attempt to represent the topological relations between the regions. The empty space between polygons allows noncontiguous cartograms to depict the shapes of the regions accurately. Olson (1976) described a construction method for noncontiguous cartograms, in which each polygon is shrunk isotropically towards its centroid. The method tends to create excessive empty space between the polygons. It can also lead to region overlaps if polygons are concave or contain holes. A fully automated fail-safe algorithm has yet to be described in the literature. Hence, the construction of noncontiguous cartograms currently relies on manual adjustments with image editing software.

Contiguous Cartograms

Contiguous cartograms correctly represent common borders and tripoints between the depicted regions (Fig. 1e). Construction methods for contiguous cartograms can be divided into two distinct classes. The first class ("boundaries-only" methods) restricts itself to moving a finite number of boundary points from their geographic position (e.g., given as latitude and longitude) to their positions on a cartogram. The second class creates a cartogram by establishing a "density-equalizing map projection" that shifts every point in continuous geographic space to a new position, regardless of whether the point is on a boundary.

Boundaries-only algorithms can influence the polygon shapes more directly than algorithms that involve continuous map projections. Consequently, several boundaries-only algorithms explicitly seek a compromise between maintaining polygon shapes and achieving correct polygon areas (House and Kocmoud 1998; Keim et al. 2004). A disadvantage of the boundaries-only approach is that it does not attribute any semantics to points that are not on a boundary. By contrast, a density-equalizing map projection makes it possible to show additional data that are resolved at a finer level than the polygons to be displayed (e.g., addresses, roads, or distributions on a fine-grained grid; see Hennig 2013).

A density-equalizing map projection is a two-dimensional function $\mathbf{T} = (\mathrm{T_x};\ \mathrm{T_y})$ that satisfies

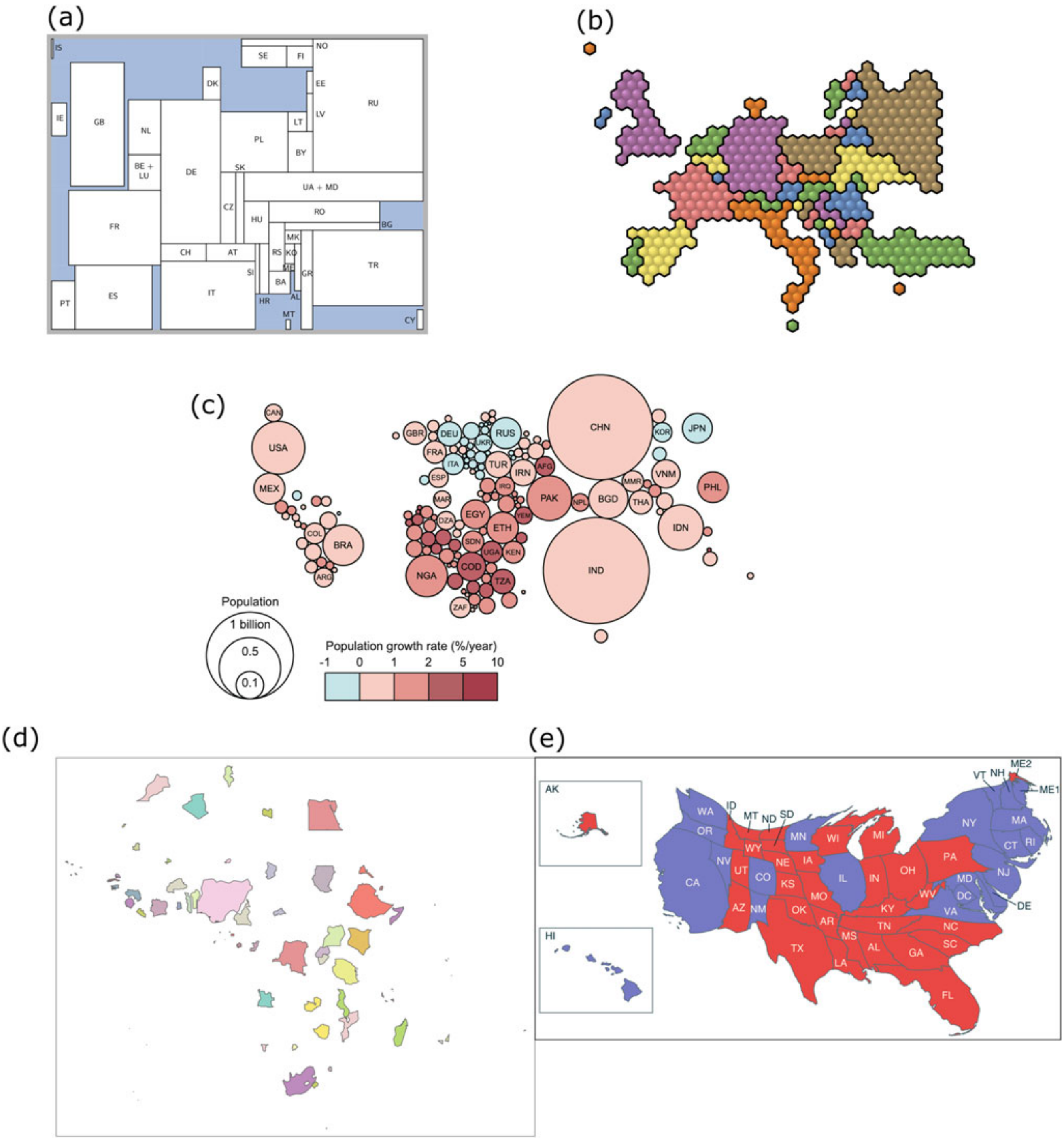
$$\det J_{\mathbf{T}} \equiv \frac{\partial \mathrm{T_x}}{\partial \mathrm{x}}\frac{\partial \mathrm{T_y}}{\partial \mathrm{y}} - \frac{\partial \mathrm{T_y}}{\partial \mathrm{x}}\frac{\partial \mathrm{T_x}}{\partial \mathrm{y}} = \frac{\rho(x,y)}{\overline{\rho}} \tag{1}$$

where det $J_{\mathbf{T}}$ is the Jacobian determinant of $\mathbf{T}$, (x, y) are coordinates on an equal-area projection, ρ(x, y) is the local density of the mapping variable (e.g., population per square kilometer), and $\overline{\rho}$ is the spatial average. Equation (1) does not uniquely specify a projection. Various additional constraints have been proposed to determine a unique solution. Tobler (1973) suggested selecting the most angle-preserving projection among all solutions to Eq. (1). In practice, Tobler's optimization algorithm failed to converge. However, several alternatives, based on analogies from physics, have led to satisfactory results (Gusein-Zade and Tikunov 1993; Sun 2020). The technique by Gastner et al. (2018) treats ρ(x, y) as the density of a fluid that equilibrates over time. This method solves Eq. (1) by calculating a vortex-free and mass-conserving velocity field. The calculation, performed with fast Fourier transforms, has been implemented as a web application (https://go-cart.io/; see Tingsheng et al. 2019).

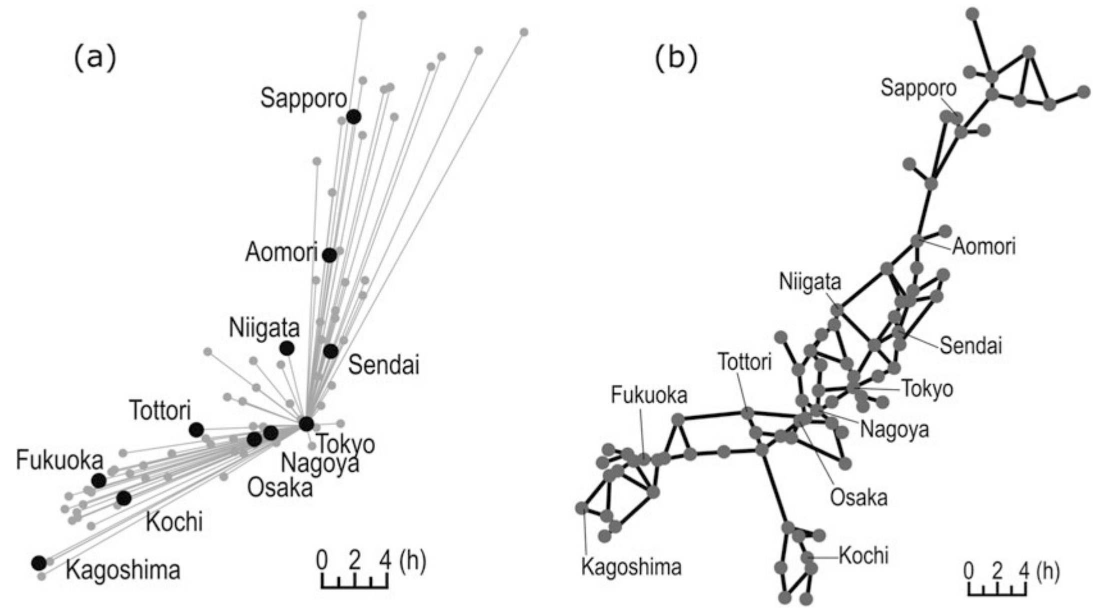
Distance Cartogram

The objective of a distance cartogram (also called "linear cartogram") is to place a set of points in a flat two-dimensional geometric space such that the distances between the points are proportional to a quantitative measure of separation. Hereinafter, we assume that the measure of separation is travel time, which is by far the most frequently used mapping variable for distance cartograms.

There are two broad categories of distance cartograms. In the first category ("central-point cartogram"), only travel



Cartogram, Fig. 1 Area cartograms. (**a**) Rectangular cartogram depicting the population of Europe (Buchin et al. 2012). (**b**) Mosaic cartogram for the same data (Cano et al. 2015). (**c**) Circular cartogram illustrating the predicted world population in 2030 (Inoue 2011). (**d**) Noncontiguous cartogram representing the 2005 population of African countries (Jeworutzki 2020). (**e**) Contiguous cartogram for the 2016 US presidential election. The area of each state in the cartogram is proportional to the number of electors. Red states were won by Republicans, blue states by Democrats (Gastner et al. 2018)



Cartogram, Fig. 2 Distance cartograms. (**a**) Central-point cartogram showing the travel time from Tokyo by train in 1965. (**b**) Network cartogram based on pairwise travel times between 81 Japanese cities. (Figure from Shimizu and Inoue (2009))

times from a central location are represented on the cartogram (Fig. 2a). The second category ("network cartograms") aims to represent travel times in a network whose links are not necessarily connected to a unique central node (Fig. 2b).

The construction of central-point cartograms is straightforward. Points can be placed at a distance that is strictly proportional to the travel time, and the angle of the connecting line to the central point can be chosen arbitrarily (e.g., to match the angle on a conventional map). Network cartograms pose greater challenges because it is generally impossible to simultaneously satisfy the distance constraints for all point pairs. Instead, the aim is to minimize the deviation from proportionality. A common objective function is

$$\min f(x_1, \ldots, x_n, y_1, \ldots, y_n) = \sum_{(i,j) \in L} \left(t_{ij} - \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \right)^2 \qquad (2)$$

where x_i and y_i are the coordinates of point i on the cartogram, and t_{ij} is the travel time from i to j. L is the set of all links whose travel times are to be visualized. Minimizing f is equivalent to multidimensional scaling, a common mathematical technique for dimension reduction in statistics. The solution to Eq. (2) is not unique because any rotation and translation leaves f unchanged. To select a unique solution, it is sensible to conduct the following post-hoc steps (Ewing and Wolfe 1977):

- Scale the cartogram coordinates such that the mean of all distances between pairs of points matches the mean interpoint distance in physical space.
- Shift the cartogram coordinates such that the centroid of the points on the cartogram is the same as the centroid of their physical locations.
- Rotate the cartogram coordinates around the centroid such that the rotated coordinates minimize the sum of the squared distances from the physical coordinates.

If L contains all $n(n - 1)/2$ possible links between n points, good numerical results were obtained using the technique described above (Marchand 1973). However, if some point pairs are missing from L, there can be multiple solutions to Eq. (2) which differ by more than an overall rotation and translation. In this case, the algorithm should aim to minimize angular distortion to produce readable cartograms (Shimizu and Inoue 2009).

It is often informative to show more than the transformed network locations $(x_1, y_1), \ldots, (x_n, y_n)$ on the cartogram. For example, coastlines and administrative boundaries can add valuable context. In this case, the transformation from the physical locations $\left(x_i^p, y_i^p\right), i \in \{1, \ldots, n\}$ to the cartogram locations (x_i, y_i) must be spread into a continuous two-dimensional space. Ideally, this task is achieved by a continuous map projection $\mathbf{T}$ that interpolates smoothly between the n points in the network such that $\mathbf{T}\left(x_i^p, y_i^p\right) = (x_i, y_i)$. However, standard interpolation methods can result in graticule cells whose winding orientation is locally inverted. That is, there can be coordinates (x,y) where the Jacobian determinant $\det J_{\mathbf{T}}(x,y)$ is negative, especially where fast long-distance connections are geographically close to slow short-distance connections. To alleviate this problem, Spiekermann and Wegener (1994) proposed a modification of multidimensional scaling, called "stepwise multi-dimensional scaling." However, a rigorous proof that this method avoids local inversions in **T** remains an open research challenge.

Summary

Conventionally, geographic maps are designed to faithfully represent metric properties such as areas, distances, or angles. Cartograms offer an alternative map design with the primary aim of visualizing quantitative statistical data instead of geometric properties. Data associated with nonoverlapping two-dimensional regions can be shown on area cartograms, for which many different designs have been developed over the past decades. Data that quantify the degree of separation between points (e.g., travel time) can be visualized with distance cartograms. Although both types of cartograms appear inevitably distorted, they can be effective alternatives to traditional thematic maps (e.g., choropleth maps or proportional-symbol maps).

Cross-References

► Data Visualization
► Fast Fourier Transform
► Optimization
► Tobler, Waldo
► Travel Time Analysis

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